Bus Q780: Mathematical Modeling in Management Science

Fall 2024 Course Outline

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Operations Management Area

McMaster University

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Contents

Abstract

Enjoy the course.

McMaster University
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- **First class is on September 9, 2024 (Monday).**
- Class times: 11:30-2:20 ([redacted])

CLASSROOM CONDUCT

NO Virtual Class!

- This course will NOT be taught virtually in Fall 2024.
- But there will be online components on Avenue which should be studied asynchronously.

Other Matters

Please respect the following line of conduct in class in order to preserve a favorable learning environment:

- Show up to class on time!
- **Phones turned off in class; no leaving class for calls!**
- **No laptop use in class!**
  - **Exception to this Rule: When I am demonstrating a software application, you can use your laptop, but once it's over, the laptop gets turned off!**

- No talking while the instructor is talking.
- Questions to be directed to the instructor.
- No reading materials unrelated to class.

1 COURSE OBJECTIVE

Majority of published (and applied) research in Management Science involves mathematical modeling using tools from stochastic processes, optimization, statistics and data science, and related disciplines. The objective of this course is to familiarize the PhD students in the Management Science field with the fundamentals of real analysis (advanced calculus), probability, linear algebra, differential equations and statistical theory in order to prepare them for the required courses in the field. The course also introduces the students to the statistical environment R with Rcmdr and the computer algebra system Maple.

2 INSTRUCTOR AND CONTACT INFORMATION

Dr. Mahmut Parlar

E-mail: ★ `http://telecom.mcmaster.ca/directory.cfm`

★ `https://parlar.azurewebsites.net/index.html`

Office: DSB-425

Office hours: TBA

3 CLASS TIME and ROOM

- **The following three classes will be cancelled, but we will have weekend make-up classes to replace them. Dates for make-up classes TBA.**
  - **September 16, 2024, Mon. (EMBA teaching)**
  - **September 23, 2024, Mon. (EMBA teaching)**
  - **October 14, 2024, Mon. (Thanksgiving)**

4 COURSE DESCRIPTION

This course will start with a presentation of the historical background of the origins of operations research and management science dating back to the Second World War. This will be followed by a discussion of the instructor's personal experience of identifying, modelling and solving problems culminating in published papers. We will then cover important tools in real analysis, probability theory, linear algebra, differential equations and statistical theory.

- Prerequisite: Enrolment in the PhD Program or permission of the instructor.

5 COURSE WEBSITE

★ (Reachable from) `http://avenue.mcmaster.ca/`

6 LEARNING OUTCOMES

Upon completion of this course, students will be able to complete the following key tasks:

- Identify real decision problems and model them
- Use advanced techniques covered in the course to solve these problems and write publishable papers
- Be familiar with mathematical proof techniques.
- Be familiar with R/Rcmdr and Maple

7 COURSE SCHEDULE

The course will cover the following topics.

7.1 Introduction and Historical Background

Classical papers.

- Queen of Dido
- Kantorovich [13]
- Second World War, Blackett's Circus: `https://en.wikipedia.org/wiki/Patrick_Blackett`
- Morse and Kimball [16]
- Operations Research 50th Anniversary Issue, Vol. 50, Number 1, January-February 2002
- Management Science 50th Anniversary Issue, Vol. 50, Number 12 Supplement, December 2004

7.2 Philosophy of Mathematical Modelling in MS/OR

Decision maker, decision variables, objective function, constraints.

- Ackoff [1, p. 32]
- Other early books by Sasieni, Yaspan and Friedman [26], and Ackoff and Sasieni [2]

7.3 How to Model a Real-life Decision Problem

My papers motivated by real-life problems.

- One-day-old cakes: Parlar [17]
- Bagel shop: Gerchak, Parlar and Yee [9]
- Competing newsboys: Parlar [18]
- Free shipping: Leng and Parlar [15]
- Elevator positioning: Parlar, Sharafali and Ou [21]
- Airline check-in counters: Parlar and Sharafali [20]
- Parking: Bakhtiari, Berk, Hassini and Parlar [4]
- And others modelling real (decision) problems.
  - h-index
  - Ponzi mathematics

7.4 Software (Maple and R/Rcmdr)

Two major software tools.

7.4.1 Maple (Parlar [19])

Interactive Operations Research with Maple

- `https://parlar.azurewebsites.net/ORMapleBook/ORMapleBook.html`

7.4.2 R/Rcmdr

Notes from 2023 EMBA course T711.

- Why R and R Commander? `https://parlar.azurewebsites.net/COURSES/emba-2023/Why-R/index.html`
- BASICS: R functions for basic statistical models with Rcmdr `https://parlar.azurewebsites.net/COURSES/emba-2023/Session-1/index.html`
- REGRESSION: R functions for multiple regression and its ramifications: `https://parlar.azurewebsites.net/COURSES/emba-2023/Session-2/index.html`

7.5 Mathematical Foundations

Basic calculus review: Derivatives and Integrals.

- Sydsæter, et al. [28]
- Osborne's web site: `https://mjo.osborne.economics.utoronto.ca/index.php/tutorial/index/1/int/i`

7.5.1 Real analysis

Weirstrass theorem with examples, Leibnitz's rule, Taylor's formula, Convexity, Jacobian, Gradient, Hessian, Implicit Function theorem, and a bit of Complex Analysis.

- R. G. Vickson's notes for MS635
- Garrity book [7]
- Peressini et al. [22]

Some discussion of convex sets and functions. Global optimization.

7.5.2 Linear algebra

Homogeneous system, Linear independence, Matrices, Determinant, Basis, Vector space, Rank, Eigenvalues and Eigenvectors (powers of matrices and PageRank), positive definiteness

- Anton [3]
- Peressini [22, p. 30]

7.5.3 Differential Equations

Modelling systems evolving over time.

ODE Savings account model as an ODE, Poisson process ODE, ODE systems, Optimal control, Wronskian

- Sydsæter, et al. [28]

PDE

- Gerchak, et al., "On Manpower Planning in the Presence of Learning", [8]
- Derzko, et al. "Dirtributed Parameter Systems Approach to the Optimal Cattle Ranching Problem," [6]
- Black-Scholes [5] PDE model of option pricing

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf, \quad f(S_T, T) = \max(S_T - K, 0)$$

  - The solution of this partial differential equation is as follows:

$$f = SN(\ell_1) - Ke^{-r(T-t)}N(\ell_2)$$

    where

$$\ell_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}},$$

$$\ell_2 = \frac{\ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}.$$

    Here, $N(\cdot)$ is the cumulative distribution of the standardized normal random variable, S is the spot price of the underlying asset, K is the exercise price, $T-t$ is the time to expiration (with t being the current time), r is the risk-free interest rate and σ is the volatility of the asset.

7.5.4 Transform techniques

Useful tools in stochastic modelling and models using differential and difference equations.

Laplace transform and its inverse,

- Heyman and Sobel [11, Appendix A.3]

Generating functions

- Heyman and Sobel [11, Appendix A.3]

7.5.5 Proof techniques and symbolic logic

You will need to learn how to prove theorems.

Proof techniques

- Snyder and Shen [27, App. B]
- Polya [23]
- Velleman [29]

Symbolic logic

- Saaty [25, pp. 51–63] (symbolic logic)
- Gustason and Ulrich [10] (symbolic logic)
- Yıldırım [31] (symbolic logic)

7.6 Probability Theory

Nature of risk/uncertainty, Joint random variables (r.v.), Functions of r.v.'s

- Ross [24]
- Mendenhall [30]
- Heyman and Sobel [11, Appendix A]
- Ramakanth [14]

7.7 Statistics and Data Science

Sampling distributions, estimation, confidence intervals, hypothesis testing, regression with vector/matrix notation. Discrimination, clustering, neural networks.

- Mendenhall [30]
- Johnson and Wichern: [12]

8 REQUIRED COURSE MATERIALS AND READINGS

8.1 Main References

The course will be based on the instructor's lecture notes. However, we will make frequent use of the books noted above and the book authored by the instructor.

- M. Parlar [19] (The use of Maple in solving non-trivial OR/MS problems).

8.2 Software Use

- We will make frequent (and substantial) use of the computer algebra system Maple (`maplesoft.com`) in this course using Parlar [19].
- We will also use R/Rcmdr
- See the next link for information and downloads on this book and related software.

  ★ `https://parlar.azurewebsites.net/ORMapleBook/ORMapleBook.html`

8.3 Suggested Reading

Books and papers in the reference list.

9 EVALUATION

9.1 Allocation of Marks

- Project involves independently identifying, modelling and solving a real-life problem.

2 Assignments	20%
Midterm Examination	30%
Project	10%
Final Examination	40%

9.2 Grade Conversion

Grade (Points)	**Percent**
A+ (12)	90–100
A (11)	85–89
A− (10)	80–84
B+ (9)	77–79
B (8)	73–76
B− (7)	70–72
F (0)	0–69

10 ACADEMIC INTEGRITY

You are expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. Academic credentials you earn are rooted in principles of honesty and academic integrity. It is your responsibility to understand what constitutes academic dishonesty.

Academic dishonesty is to knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage. This behaviour can result in serious consequences, e.g. the grade of zero on an assignment, loss of credit with a notation on the transcript

(notation reads: "Grade of F assigned for academic dishonesty"), and/or suspension or expulsion from the university.

For information on the various types of academic dishonesty please refer to the Academic Integrity Policy, located at https://secretariat.mcmaster.ca/university-policies-procedures-guidelines/

The following illustrates only three forms of academic dishonesty:

- plagiarism, e.g. the submission of work that is not one's own or for which other credit has been obtained.
- improper collaboration in group work.
- copying or using unauthorized aids in tests and examinations.

11 MISSED ACADEMIC WORK

Late assignments will not be accepted. No extensions are available except under extraordinary circumstances. Please discuss any extenuating situation with your instructor at the earliest possible opportunity.

12 STUDENT ACCESSIBILITY SERVICES

Student Accessibility Services (SAS) offers various support services for students with disabilities. Students are required to inform SAS of accommodation needs for course work at the outset of term. Students must forward a copy of such SAS accommodation to the instructor normally, within the first three (3) weeks of classes by setting up an appointment with the instructor. If a student with a disability chooses NOT to take advantage of an SAS accommodation and chooses to sit for a regular exam, a petition for relief may not be filed after the examination is complete. The SAS website is:

★`http://sas.mcmaster.ca`

13 ACADEMIC ACCOMMODATION FOR RELIGIOUS, INDIGENOUS OR SPIRITUAL OBSERVANCES (RISO)

Students requiring academic accommodation based on religious, indigenous or spiritual observances should follow the procedures set out in the RISO policy. Students should submit their request to their Faculty Office normally within 10 working days of the beginning of term in which they anticipate a need for accommodation or to the Registrar's Office prior to their examinations. Students should also contact their instructors as soon as possible to make alternative arrangements for classes, assignments, and tests.

14 POTENTIAL MODIFICATIONS TO THE COURSE

The instructor and university reserve the right to modify elements of the course during the term. The university may change the dates and deadlines for any or all courses in extreme circumstances. If either type of modification becomes necessary, reasonable notice and communication with the students will be given with explanation and the opportunity to comment on changes. It is the responsibility of the student to check their McMaster email and course websites weekly during the term and to note any changes.

References

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[5] F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81:637–659, May–June 1973.

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[20] M. Parlar and M. Sharafali. Dynamic allocation of airline check-in counters: A queueing optimization approach. *Management Science*, 54(8):1410–1424, August 2008.

[21] M. Parlar, M. Sharafali, and J. Ou. Optimal parking of idle elevators under myopic and state-dependent policies. *European Jurnal of Operational Research*, 170(3):863–886, 2006.

[22] A. L. Peressini, F. E. Sullivan, and J. J. Uhl. *The Mathematics of Nonlinear Programming*. Springer-Verlag, New York, 1988.

[23] G. Polya. *How to solve it: A new aspect of mathematical method*. Princeton University Press, 2nd edition, 1985. First published in 1944.

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